1. The angle $\angle DEF = \cos^{-1}\left(\frac{DE^2 + EF^2 - DF^2}{2\,DE \times EF}\right)$

$= \cos^{-1}\left(\frac{7.1^2 + 10^2 - 9.7^2}{2 \times 7.1 \times 10}\right)$

$= 66.63^\circ$

2. We know the area of a triangle

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where a, b, c are three sides and $s = \frac{a+b+c}{2}$

Putting $A = 15\ cm^2$; $a = 15\ cm$, $b = 4\ cm$, we find,

$c = 11.708\ cm$ or $18.572\ cm$

The angle θ is given by $\cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right)$

$= 30^\circ$ or 150°

3. Lets the three sides be a, b & c. Here $a = 42\,m$, $b = 51\,m$ and $\theta = 68°$

we know, $\cos\theta = \dfrac{a^2+b^2-c^2}{2ab}$

We find $c = \sqrt{a^2+b^2-2ab\cos\theta} = 52.538\ m$

Now using $\dfrac{a}{\sin\angle A} = \dfrac{b}{\sin\angle B} = \dfrac{c}{\sin\theta}$

$$B = \sin^{-1}\left(\frac{b}{c}\sin\theta\right) = 64.164°$$

$$A = \sin^{-1}\left(\frac{a}{c}\sin\theta\right) = 47.837°$$

4. We find the angle $\angle RST = 180^\circ - 23^\circ - 97^\circ$

$= 60^\circ$

Now we use the following formula :

$$\frac{RT}{\sin(\angle RST)} = \frac{RS}{\sin(\angle STR)}$$

or, $$RT = RS \times \frac{\sin(\angle RST)}{\sin(\angle STR)}$$

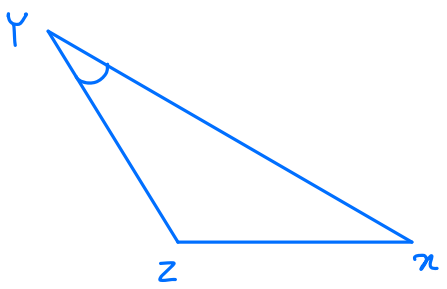
$$= 12 \times \frac{\sin 60^\circ}{\sin 97^\circ} = 10.470$$

5. $3\tan^2\theta - 1 = 0$ or, $\tan\theta = \pm \frac{1}{\sqrt{3}}$

Taking $\tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = -150^\circ, 30^\circ$

taking $\tan\theta = -\frac{1}{\sqrt{3}} \Rightarrow \theta = -30^\circ, 150^\circ$

6.



We know

$$\frac{|zx|}{\sin\angle Y} = \frac{|YZ|}{\sin\angle x}$$

so, $$\angle x = \sin^{-1}\left[\frac{|YZ|}{|zx|}\sin\angle Y\right]$$

$$\text{or, } \angle x = \sin^{-1}\left[\frac{4.73}{3.70} \times \sin 42^\circ\right]$$

$$= 58.804$$

$$\therefore \angle z = 180^\circ - \angle x - \angle y = 180 - 58.804 - 42^\circ$$

$$= 79.196^\circ$$

Lets take $|yz| = 3.70$ & $|zx| = 4.73$

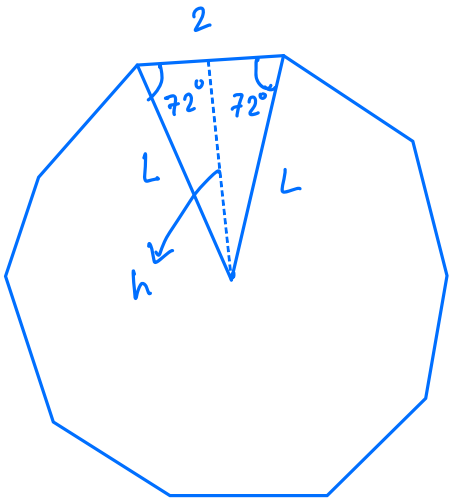
then $$\angle x = \sin^{-1}\left[\frac{|yz|}{|zx|} \sin \angle y\right]$$

$$\text{or, } \angle x = \sin^{-1}\left[\frac{3.70}{4.73} \times \sin 42^\circ\right]$$

$$= 31.562^\circ$$

$$\angle z = 180^\circ - 42^\circ - 31.562^\circ = 106.438^\circ$$

(7.)



From the figure we find,

$L \cos 72^\circ = 1$

or, $L = \frac{1}{\cos 72^\circ}$

and $h = L \sin 72^\circ$

$= \tan 72^\circ$

$\therefore$ The area of the triangle $= \frac{1}{2} \times 2 \times h$

$= \tan 72^\circ$

$\therefore$ The area of the decagon $= 10 \tan 72^\circ$

$= 30.777 \text{ unit}^2$