1. Given that, $3x + 5 \leq 10 - x$

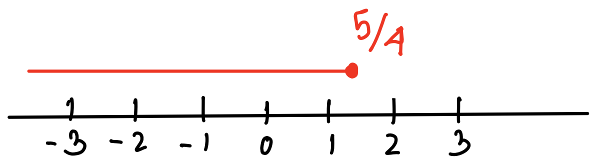
or, $4x \leq 5$

or, $x \leq \frac{5}{4}$

set descriptor notation : $\left\{ x \mid x \leq \frac{5}{4} \right\}$

interval notation : $\left(-\infty, \frac{5}{4}\right]$

Graph:



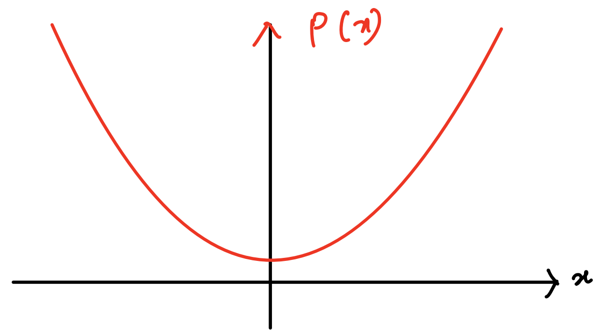
2. Statements that are true.

A. $|x+4|$ is the distance between x & -4. True

B. $x^2 + x + 2 < 0$ True

we plot the curve $P(x) = x^2 + x + 2$.

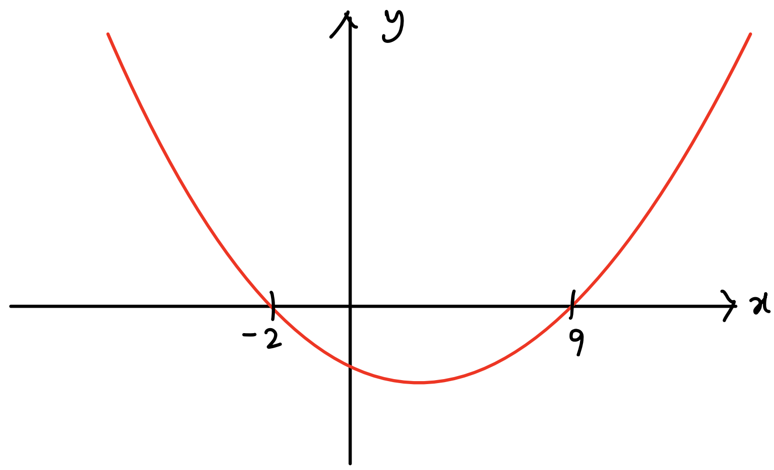
We observe that $P(x)$ never crosses the x axis. So, there is no solution.



C. $\frac{2x+1}{x-3} = 0$ when $2x + 1 = 0$ False

D. The values $x = 8, 20$ do not belong to the solution set. False

3.



$$y = x^2 - 7x - 18$$
$$= (x+2)(x-9)$$

From the plot we observe that, $y \leq 0$ when $x \in [-2, 9]$

or, $x^2 - 7x - 18 \leq 0$

or, $x^2 - 7x \leq 18$ when $x \in [-2, 9]$

4. (A) $\{x \mid x < 5 \text{ and } x < -2\}$

Note $x < -2$ implies $x < 5$. Hence it can be written as

$\{x \mid x < -2\}$

(B). $\{x \mid x > -5 \text{ and } x < -2\}$

$= \{x \mid -5 < x < -2\}$

(C). $\{x \mid x > 5 \text{ and } x < -2\} = \{\}$

as the two inequalities can not be satisfied simultaneously.

(D). $\{x \mid x > 5 \text{ and } x > -2\} = \{x \mid x > 5\}$

as $x > 5$ implies $x > -2$

5. $x^2 - 3x \leq 4$ or, $x^2 - 3x - 4 \leq 0$

or, $(x-4)(x+1) \leq 0$

either $x - 4 \geq 0$ & $(x+1) \leq 0$

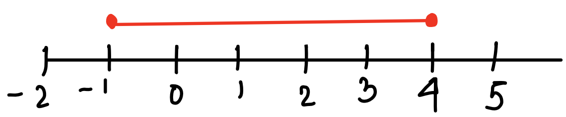
or, $x \geq 4$ & $x \leq -1$ not possible.

the other solution : $(x-4) \leq 0$ & $(x+1) \geq 0$

or, $x \leq 4$ & $x \geq -1$

- Set descriptor notation : $\{ x \mid -1 \leq x \leq 4 \}$
- interval notation : $x \in [-1, 4]$

Graph :



6. Given that, $\dfrac{x-5}{3x+2} \geq 0$

one solution, $x - 5 \geq 0$ & $3x + 2 > 0$

or, $x \geq 5$ & $x > -2/3$

which implies $x \geq 5$

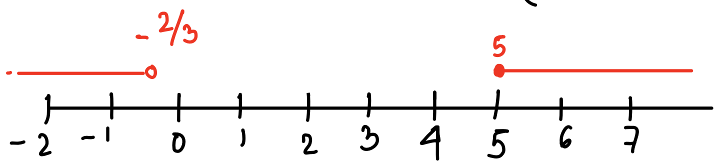
the other solution: $x - 5 \leq 0$ & $3x + 2 < 0$

or, $x \leq 5$ & $x < -2/3$

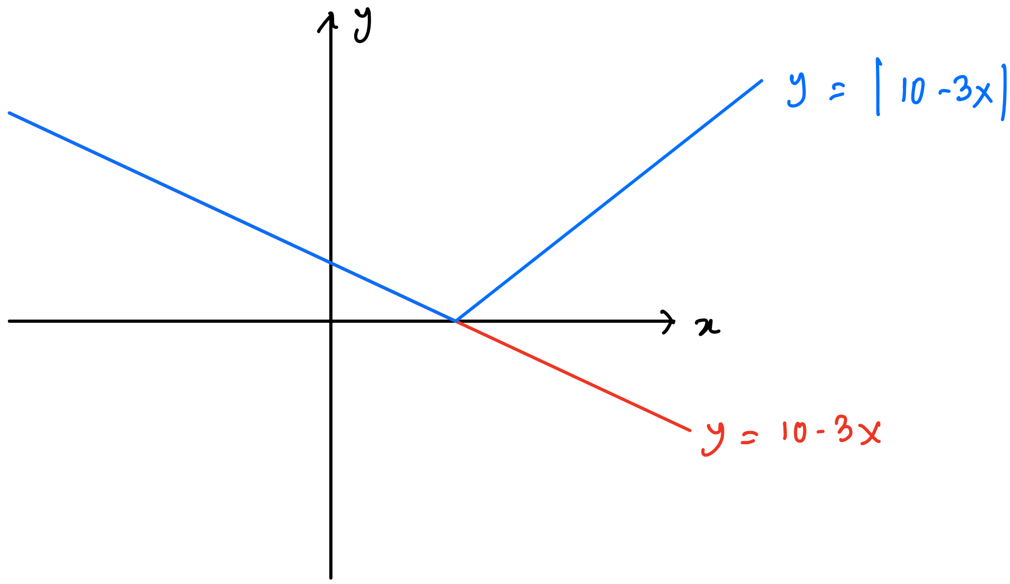
which implies $x < -2/3$

∴ The solution is $\{x \mid x < -2/3 \text{ or } x \geq 5\}$

$x \in \left(-\infty, -\frac{2}{3}\right) \cup [5, \infty)$



7.



8. $|a| \leq 5$

when $a > 0$: $a \leq 5$

$a < 0$: $-a \leq 5$ or, $a \geq -5$

so. $-5 \leq a \leq 5$ (option B)

9. $|6x + 12| \geq 18$

: $6x + 12 \geq 18$ or, $x \geq 1$

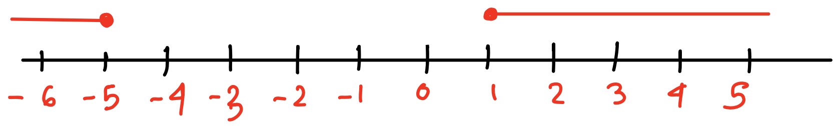
$\Rightarrow x \geq 1$

: $-(6x + 12) \geq +18$ or, $x \leq -5$

$\Rightarrow x \leq -5$

$\therefore$ The solution: $\{x \mid x \leq -5 \text{ or } x \geq 1\}$

or, $x \in (-\infty, -5] \cup [1, \infty)$



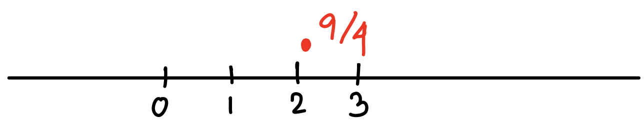
(10) $|4x - 9| \leq 0$

so we have $4x - 9 \leq 0$ or, $x \leq \frac{9}{4}$

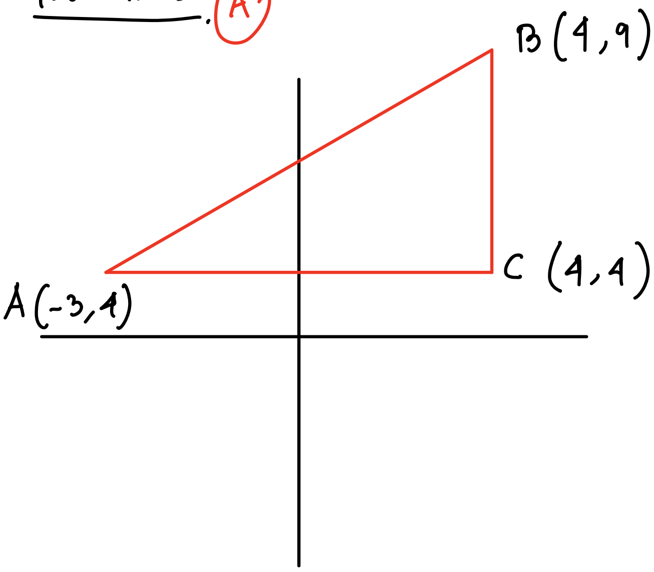
and $-(4x - 9) \leq 0$ or, $4x - 9 \geq 0$ or, $x \geq 9/4$

The solution is: $\{x \mid x = \frac{9}{4}\}$

$x \in \{\frac{9}{4}\}$



Problem 1 (A)



- $C = (4,4)$
- $AC = 4-(-3) = 7$

  $BC = 9-4 = 5$
- $AB = \sqrt{AC^2 + BC^2}$

  $= \sqrt{7^2+5^2}$

  $= \sqrt{49+25} = \sqrt{74}$

- The distance formula is $\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$

$$= \sqrt{(4+3)^2 + (9-4)^2}$$

$$= \sqrt{7^2 + 5^2} = \sqrt{74}$$

- Both the formula are same.

(B) The distance between DE $= \sqrt{(7-(-5))^2 + (3-(-2))^2} = \sqrt{12^2+5^2} = 13$

The midpoint $M = \left(\frac{7-5}{2}, \frac{3-2}{2}\right) = \left(1, \frac{1}{2}\right)$

(C) The distance of DM $= \sqrt{(-5-1)^2 + \left(-2-\frac{1}{2}\right)^2} = \sqrt{6^2 + \frac{5^2}{2^2}} = \frac{13}{2}$

the distance ME $= \sqrt{(7-1)^2 + \left(3-\frac{1}{2}\right)^2} = \sqrt{6^2+\frac{5^2}{2^2}} = \frac{13}{2}$

$\therefore$ DM = ME = $\frac{DE}{2}$ as it should be.

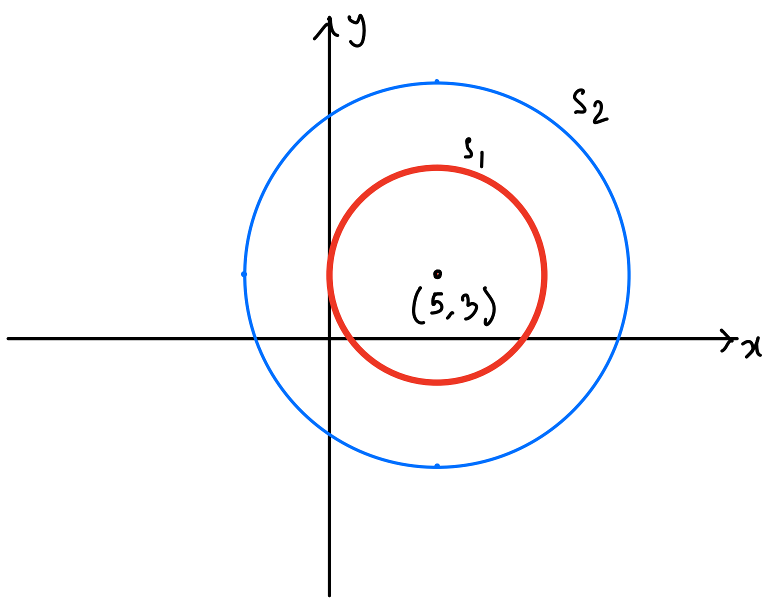
D. The eqn of the line containing D $(-5,-2)$ and E $(7,3)$ is:

$$\frac{y-3}{x-7} = \frac{3+2}{7+5}$$

or $(y-3) = \frac{5}{12}(x-7)$

Problem 2

A. $S_1 := (x-5)^2 + (y-3)^2 = 25$ and $S_2 : (x-5)^2 + (y-3)^2 = 81$



These two circles are concentric.

B. The eqn of the circle is: $(x+2)^2 + y^2 = 6$

C. unit circle $x^2+y^2=1$

D. The center is at $M : \left(\frac{50-10}{2}, \frac{84+16}{2}\right) = (20, 50)$

and radius $R = \frac{1}{2}\sqrt{(50+10)^2 + (84-16)^2} = 2\sqrt{514}$

$\therefore$ The eqn is $(x-20)^2 + (y-50)^2 = 4 \times 514 = 2056$

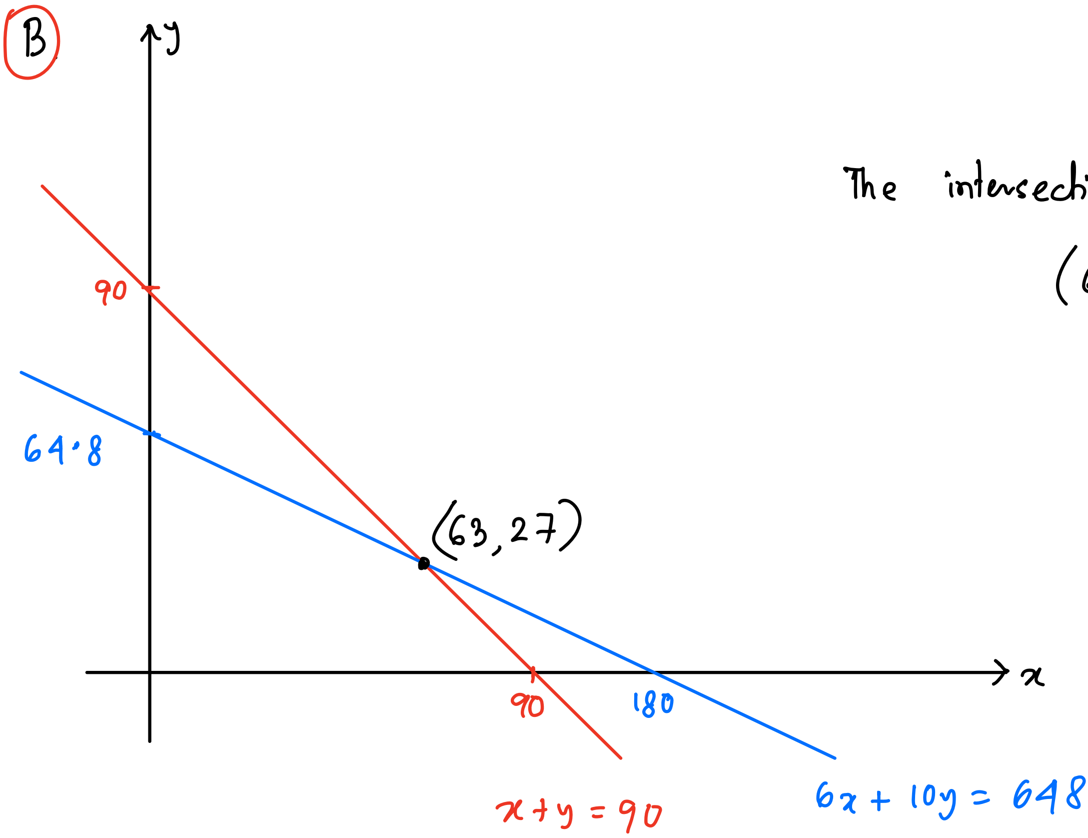
Problem 3

(A) The two eqns are $x + y = 90$ or, $\frac{x}{90} + \frac{y}{90} = 1$

and $6x + 10y = 648$ or, $\frac{x}{108} + \frac{y}{324/5} = 1$

(B)

The intersection point is $(63, 27)$



The number of thingamabobs is 63

& number of whatchamacallits is 27

(C) $x + y = 90 \Rightarrow y = 90 - x$

$6x + 10y = 648 \Rightarrow y = \frac{1}{10}(648 - 6x)$

so, $90 - x = \frac{1}{10}(648 - 6x)$

or, $900 - 10x = 648 - 6x$

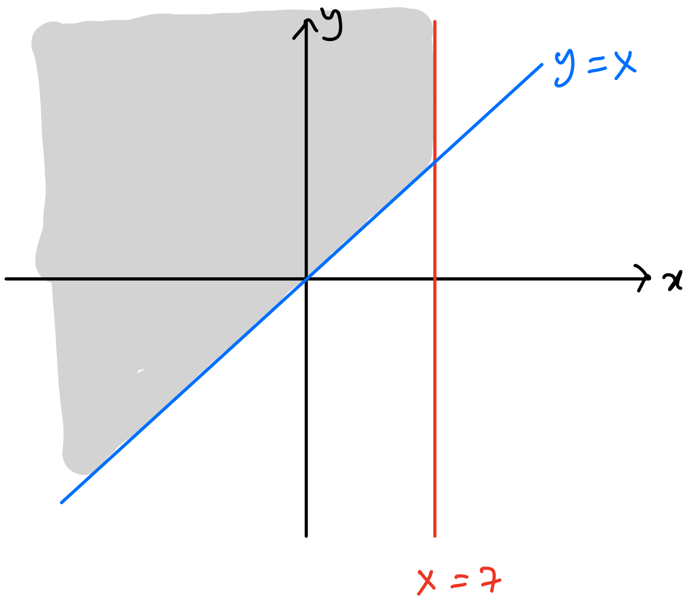
or, $4x = 900 - 648 = 252$

or, $x = 63$

So, $y = 90 - x = 90 - 63 = 27$

Problem 4.

(A) $x \leq 7$ & $y \geq x$



option 4

(B) $2x - 3y \leq 12$ & $x + 2y \leq 4$